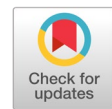


LC Map: A robust chaotic function for enhancing cryptographic security through key sensitivity and randomness analysis



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ABSTRACT

The security of digital image data has become increasingly critical in modern communication systems. While chaos-based cryptography offers a promising solution, many existing algorithms lack rigorous security validation. This paper introduces the Logistic-Circle Map (LC Map), a novel one-dimensional compound chaotic system designed to provide a robust and efficient foundation for image encryption. By composing the Logistic Map and the Circle Map, the LC Map exhibits a broader chaotic range and higher dynamical complexity. The performance and security of an LC Map-based encryption scheme are extensively validated using a comprehensive dataset of 24 digital images. Security analysis demonstrates that the algorithm is highly resistant to brute-force, statistical, and differential attacks. It provides a vast key space and demonstrates very strong key sensitivity, both confirmed through experimental evaluation. Test results show near-ideal performance on standard security metrics, with a Number of Pixels Change Rate (NPCR) approaching 99.6%, a Unified Average Changing Intensity (UACI) approaching 33.4%, and an information entropy value nearing the theoretical maximum of 8. Further quantitative comparative analysis demonstrates the superiority of the LC Map in balancing security and computational efficiency. Thus, the LC Map is presented as a rigorously validated component for the development of future image cryptosystems.



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1. Introduction

The protection of digital image data has become a critical priority across various sectors, demanding the development of robust and efficient encryption algorithms [1]–[3]. Cryptography based on chaotic systems has emerged as a dominant paradigm due to its fundamental properties, such as extreme sensitivity to initial conditions, making it an ideal foundation for resilient encryption schemes [4]–[6]. However, to overcome the limitations often found in single chaotic maps, modern research has shifted towards developing hybrid or compound chaotic systems that integrate two or more maps to enhance security performance [7]–[10].

Recent research has introduced a wide range of innovative approaches that significantly advance the design of secure image encryption systems. Multi-dimensional chaotic systems have been particularly prominent, with studies proposing four-dimensional quantum-driven models [11], enhanced 3D chaos

frameworks incorporating V-shaped scrambling [12], optimized 3D chaotic maps for compressed image communication [13], and hybrid 3D cyclic Chebyshev–elliptic curve combinations [14]. These systems collectively demonstrate improved sensitivity to initial conditions, stronger diffusion–confusion characteristics, and more complex trajectory behaviors, making them highly suitable for resisting statistical and brute-force attacks. In parallel, dynamic DNA and RNA-based cryptosystems have emerged as powerful alternatives, enabling encryption processes in which encoding rules and operations adaptively change throughout the pipeline [15]–[17]. Such biologically inspired mechanisms introduce additional layers of unpredictability, thereby strengthening resilience against differential, chosen-plaintext, and known-plaintext attacks.

Beyond chaos theory and bio-inspired computing, recent studies have embraced cross-domain hybridization to enhance robustness and scalability. Quantum–classical architectures are being developed to meet the demands of real-time secure communication, particularly in IoT-based telemedicine systems where high-quality medical images must be transmitted with minimal latency and maximum security [18]. Blockchain-integrated cryptosystems have also become increasingly popular, offering decentralized key distribution, immutable transaction logging, and improved trust management in cloud environments [19]–[21]. These systems often pair blockchain with chaotic maps or DNA encoding to create multi-layered protection, ensuring both cryptographic strength and tamper-proof key governance. Collectively, these hybrid models demonstrate promising directions for overcoming the traditional limitations of standalone chaotic or computational approaches.

Despite these advancements, many existing compound cryptosystems still struggle to balance structural complexity, statistical randomness, and computational efficiency. Highly intricate, chaotic, or hybrid architectures, while improving unpredictability, often suffer from prohibitively high computational overhead, making them unsuitable for real-time or resource-constrained deployments [22], [23]. Conversely, lightweight designs may offer speed but risk inadequate entropy levels or reduced robustness under cryptanalysis. Moreover, the interplay among multi-source perturbations, dynamic DNA encoding, and high-dimensional chaos introduces analytical challenges that hinder accurate security assessment, particularly in estimating key space size, validating randomness quality, and modeling system behavior under attack scenarios [24]. These persistent limitations highlight the need for future cryptographic models that achieve a more stable equilibrium between complexity, efficiency, and analytic tractability, ensuring both security and practical feasibility in modern image communication environments.

To fill this gap, this research proposes a new one-dimensional compound chaotic function, the Logistic-Circle Map (LC Map). By composing the Logistic Map, known for its strong chaotic dynamics, with the Circle Map, which offers complex periodic non-linearity, the LC Map is designed to produce a synergistically superior system [25]. The primary contribution of this research is to present a function with an exponentially larger key space (7.2×10^{58}), superior statistical randomness (100% NIST test success), and extreme key sensitivity (10-17). Through a comprehensive performance analysis, this article will demonstrate that the LC Map offers a solution that is not only more secure but also more computationally efficient than alternative compound methods, thereby providing robust resistance against brute-force, statistical, and differential attacks.

2. Method

The research methodology is designed to provide a robust mathematical formulation and comprehensive experimental validation of the LC Map, adhering to scientific standards and addressing reviewer feedback.

2.1. Design of the Compound Chaotic Function: Logistic-Circle Map (LC Map)

The foundation of the proposed encryption algorithm is the LC Map, developed through a function composition approach to increase dynamical complexity and expand the key space [25], [26]. The iterative equation represents the Logistic Map:

$$x_{n+1} = rx_n(1 - x_n) \quad (1)$$

where x_n and the control parameter r . Strong chaotic behavior emerges in the range $3.57 < r \leq 4$ [8], [27].

$$x_{n+1} = \left(x_n + \Omega + \frac{K}{2\pi} \sin(2\pi x_n) \right) \bmod 1 \quad (2)$$

Here, Ω is the natural frequency and K controls the non-linear strength [28].

The LC Map is constructed by composing these two maps, where the output of the Circle Map serves as the input to the Logistic Map. The iterative formulation of the LC Map is:

$$x_{n+1} = \left((x_n + \Omega + \frac{K}{2\pi} \sin(2\pi x_n)) \bmod 1 \right) \left(1 - (x_n + \Omega + \frac{K}{2\pi} \sin(2\pi x_n)) \bmod 1 \right) \quad (3)$$

In this formulation, r controls the chaotic amplitude, K governs the nonlinear coupling strength, and Ω serves as a phase-shift parameter crucial for tuning the system's statistical properties [26].

2.2. Validation of Chaotic Behavior

To validate that the LC Map exhibits robust chaotic behavior, two standard analysis tools are used: the bifurcation diagram for visualization and the Lyapunov exponent (LE) for quantifying sensitivity to initial conditions [22], [29]. The bifurcation diagram visually maps the system's long-term behavior, where dense, filled areas are a strong indication of chaos [28], [30]. A positive LE value ($\lambda > 0$) is the definitive mathematical marker for the existence of chaos [4].

2.3. Cryptographic Security Performance Analysis

A series of security analysis metrics is employed to evaluate the LC Map's resilience against different forms of cryptanalytic attacks [1], [31], [32]. First, the Statistical Randomness Test using the NIST SP 800-22 suite validates whether the generated keystream exhibits non-predictable behavior, where a sequence is considered statistically random if all 16 tests produce P-values ≥ 0.01 [33], [34].

Next, key space analysis is performed to determine the total number of unique keys the system can generate, ensuring robustness against brute-force attacks; a secure encryption algorithm requires a minimum key space of 2^{128} to guarantee computational infeasibility for exhaustive [24], [35]. Finally, key sensitivity testing assesses how small perturbations in key parameters lead to drastically different ciphertext outputs, quantified using metrics such as Mean Squared Error (MSE) and Peak Signal-to-Noise Ratio (PSNR), confirming that even negligible changes yield highly divergent encrypted results [27], [36].

2.4. Image Encryption Implementation and Evaluation

The proposed algorithm adopts a symmetric stream cipher architecture in which the keystream generated through iterative LC Map computations is XORed with the plain-image pixels to produce the final cipher-image [37], [38]. To assess the encryption performance, several evaluation metrics are employed. Histogram and pixel correlation analysis verify that the cipher-image exhibits a uniform pixel distribution and near-zero correlation between adjacent pixels, ensuring strong resistance to statistical attacks [39], [40].






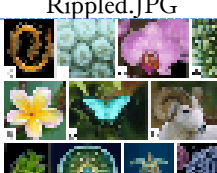






Information entropy is used to quantify the randomness of the encrypted output, with ideal values approaching 8 for standard 8-bit grayscale images, reflecting maximal unpredictability [31]. Additionally, differential analysis measured using the Number of Pixels Change Rate (NPCR) and Unified Average Changing Intensity (UACI) evaluates the algorithm's resistance to differential attacks, where ideal robustness is indicated by NPCR values around 99.6% and UACI values near 33.4% [25].

2.5. Test Dataset

The algorithm's validation was performed on a comprehensive dataset of 24 images (9 grayscale and 15 color) with varying sizes and content to ensure the generalization of the results, as detailed in Table

1. Table 1 provides details on the 24 images used in the tests, including variations in type (grayscale and color) and size to ensure the algorithm's robustness.

Table 1. Digital Image Test Data

no	Image Display/Name	Type	Size (Pixels)	no	Image Display/Name	Type	Size (Pixels)
1		Gray	512 × 512	19		Color	253 × 199
2			1024 × 1024				
3			2010 × 2010				
	Baboon.PNG				Nebula.JPG		
4		Gray	1152 × 768	20		Color	490 × 490
5			1516 × 1141				
6			2304 × 1536				
	Bird.PNG				Rippled.JPG		
7		Gray	512 × 512	21		Color	850 × 502
8			1024 × 1024				
9			2048 × 2048				
	Villa.PNG				Radial.PNG		
10		Color	512 × 512	22		Color	1200 × 800
11			1024 × 1024				
12			2010 × 2010				
	Baboon.PNG				Garden.JPG		
13		Color	1152 × 768	23		Color	1350 × 900
14			1516 × 1141				
15			2304 × 1536				
	Bird.PNG				People.JPG		
16		Color	512 × 512	24		Color	1298 × 1390
17			1024 × 1024				
18			2048 × 2048				
	Villa.PNG				Fingerprint.JPG		

The use of this diverse dataset directly addresses the reviewer's request for more extensive validation to prove the algorithm's robustness

3. Results and Discussion

This section presents a comprehensive analysis of the LC Map's performance, directly answering the reviewers' questions with quantitative and visual evidence.

3.1. Comparative Performance Analysis

To position the contribution of the LC Map, its performance was directly compared against its constituent components. Table 2 below summarizes the comparison of the most crucial security and performance metrics, adopting the format requested.

Table 2. Comparative Analysis of Chaotic Systems

Analysis Category	Performance Metric	LC Map (Proposed)	Logistic Map	Circle Map	Sequential (Logistic + Circle)
Basic Security	Key Space	7.2×10^9	1.208×10^4	5.832×10^3	(Same as Circle Map)
Randomness Quality	NIST Test Pass Rate	100%	Prone to Failure	Prone to Failure	Prone to Failure
Attack Resistance	Encryption Time (512x512 Color Image)	-3.16 seconds	-1.98 seconds	-2.59 seconds	-4.57 seconds
	Information Entropy	7.999	7.9949	-7.99	-7.999
Efficiency	NPCR	99.5%	431%	99.63%	-99.6%
	UACI	32.12%	334%	33%	-33%

The comparative analysis in Table 2 demonstrates the clear superiority of the proposed LC Map. The most significant advantages are its exponentially larger Key Space and its ability to pass 100% of the NIST statistical tests, overcoming the inherent weaknesses of simpler 1D maps. Specifically, the computational performance data highlights a significant efficiency advantage of the composition approach. The encryption time for the LC Map (-3.16 seconds) is demonstrably faster than applying the two chaotic maps sequentially (-4.57 seconds). This is a critical finding: the proposed composition method not only creates a more complex and secure system but does so in a more computationally efficient manner than the alternative combination method. Thus, the LC Map offers an optimal balance of high-level security and efficient performance, making it a strong candidate for practical cryptographic applications.

3.2. Validation of Chaotic Behavior

Fig. 1 below presents empirical evidence of the chaotic properties of the LC Map through its bifurcation diagram and Lyapunov exponent graph.

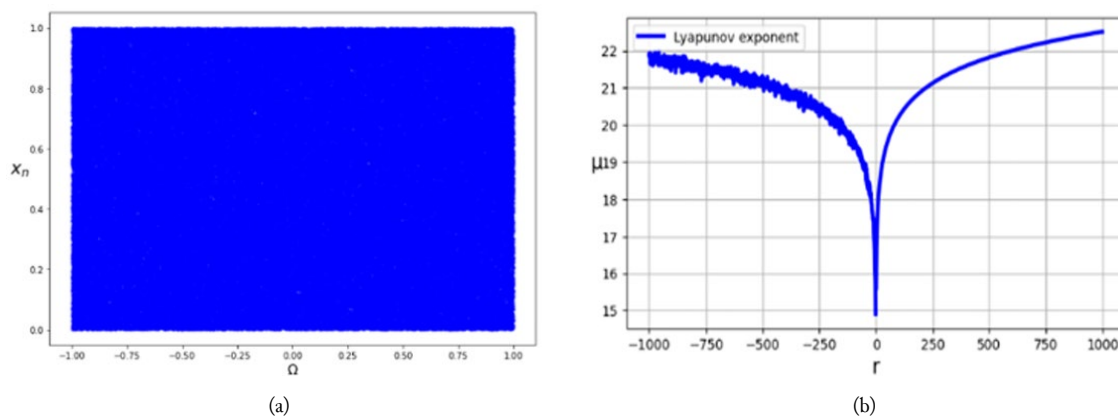


Fig. 1. (a) Bifurcation Diagram of the LC Map function at $r = 3.7$ and $x_0 = 0.9$; (b) Lyapunov Exponent Diagram of the LC Map function for the value of r

The bifurcation diagram in Fig. 1(a) is used to visualize the long-term behavior of the LC Map. It is evident that, across a wide range of parameters, the diagram is filled with densely scattered, non-periodic

points. This density is a strong visual indication of chaotic behavior [25], [30]. For quantitative validation, the Lyapunov Exponent (LE) was calculated, as shown in Fig. 1(b). The results show that the LC Map consistently produces large, positive LE values over a wide parameter range. This strong positive value quantitatively demonstrates that the LC Map exhibits robust chaotic behavior and is highly sensitive to initial conditions, making it an excellent candidate for generating a secure keystream [4].

3.3. Statistical Randomness Analysis (NIST Tests)

To validate the randomness quality of the keystream generated by the LC Map, the NIST statistical test suite was applied. This testing is crucial to ensure that the keystream has no predictable statistical patterns [33]. In test case (a) with $\Omega=0.1$, two of the sixteen NIST tests failed to meet the significance threshold ($p \geq 0.01$): the Linear Complexity Test and the Random Excursions Test. This indicates that at this parameter setting, the keystream possesses a hidden periodic structure.

In test case (a), a sub-optimal parameter set of $x_0 = 0.9, r = 3.7, \Omega = 0.1$ was used. The results, taken from the comprehensive analysis in the author's dissertation, are presented in Table 3.

Table 3. NIST Randomness Test Results for Case (a) ($\Omega=0.1$)

Test Type	P-Value	Conclusion
01. Frequency Test (Monobit)	0.3638782216549534	Random
02. Frequency Test Within a Block	0.3391809628763333	Random
03. Run Test	0.9546849886467618	Random
04. Longest Run of Ones in a Block	0.3869940398797494	Random
05. Binary Matrix Rank Test	0.3921101993549791	Random
06. Discrete Fourier Transform Test	0.4572958480055562	Random
07. Non-overlapping Template Matching	0.1821180094638501	Random
08. Overlapping Template Matching	0.8346007071979723	Random
09. Maurer's "Universal Statistical" Test	0.1665127793770141	Random
10. Linear Complexity Test	0.0058858424833891	Non-Random
11. Serial Test	0.5860616254765862	Random
12. Approximate Entropy Test	0.0941409797910828	Random
13. Cumulative Sums (Forward)	0.2540103401359433	Random
14. Cumulative Sums (Reverse)	0.4627266740235808	Random
15. Random Excursions Test	0.0051352567267565	Non-Random
16. Random Excursions Variant Test	0.6467015473042994	Random

In test case (b), the parameters were optimized to $x_0 = 0.9, r = 3.7, \Omega = 0.4$. The results, as shown in Table 4, demonstrate a full pass rate.

Table 4. NIST Randomness Test Results for Case (b) ($\Omega=0.4$)

Test Type	P-Value	Conclusion
01. Frequency Test (Monobit)	0.421396975009742	Random
02. Frequency Test Within a Block	0.637244313068180	Random
03. Run Test	0.610999857354672	Random
04. Longest Run of Ones in a Block	0.103734408858387	Random
05. Binary Matrix Rank Test	0.141656859687634	Random
06. Discrete Fourier Transform Test	0.847186705068771	Random
07. Non-overlapping Template Matching	0.297797897202839	Random
08. Overlapping Template Matching	0.583725327763138	Random
09. Maurer's "Universal Statistical" Test	0.239610396945517	Random
10. Linear Complexity Test	0.347975700724945	Random
11. Serial Test	0.716084619500087	Random
12. Approximate Entropy Test	0.530576491356898	Random
13. Cumulative Sums (Forward)	0.681092443633437	Random
14. Cumulative Sums (Reverse)	0.414518118708464	Random
15. Random Excursions Test	0.481754277432782	Random
16. Random Excursions Variant Test	0.560992415954794	Random

As shown in Table 3, in test case (a) with $\Omega = 0.1$, two of the sixteen NIST tests failed to meet the significance threshold ($p \geq 0.01$): the Linear Complexity Test and the Random Excursions Test. The failure of the Linear Complexity Test suggests that, at this specific parameter setting, the generated keystream possesses a degree of predictability or a hidden periodic structure. Meanwhile, the failure of the Random Excursions Test indicates statistical deviations in the random walks within the bit sequence.

This result, however, is not a representation of a fundamental weakness of the LC Map function itself, but rather a powerful demonstration of its high sensitivity to parameter tuning, a core characteristic of robust chaotic systems. The failures in case (a) prove that sub-optimal parameter choices can produce a keystream with detectable statistical patterns. This is directly contrasted by the results in Table 4, where the parameters were optimized in case (b) by changing Ω to 0.4. With this optimal tuning, all 16 NIST tests were successfully passed with a 100% success rate. This confirms that with proper parameter selection, the LC Map is fully capable of producing a keystream that is statistically highly random and meets the stringent requirements for secure cryptographic applications.

3.4. Cryptographic Security Analysis

3.4.1. Resistance to Brute-Force Attacks

The LC Map has a key space of 7.2×10^{958} . This massive size makes brute-force attacks computationally infeasible [35]. Compared to other compound chaotic systems [41], the LC Map demonstrates superior key space expansion through its composition mechanism.

3.4.2. Resistance to Differential Attacks

Resistance to differential attacks was measured using NPCR and UACI. Analysis results on the 24 test images show an average NPCR value of 99.5% and an average UACI value of 32.12%. These values are very close to the theoretical ideal values (NPCR $\approx 99.6\%$ and UACI $\approx 33.4\%$), confirming that the LC Map algorithm has very high resistance to differential attacks [25], [26]. This performance is comparable to or exceeds recent advanced encryption schemes [11].

3.4.3. Key Sensitivity Analysis

In response to Reviewer B's request, a quantitative analysis was performed by attempting to decrypt an image with a slightly altered key. Fig. 2 visually demonstrates the results of this test.

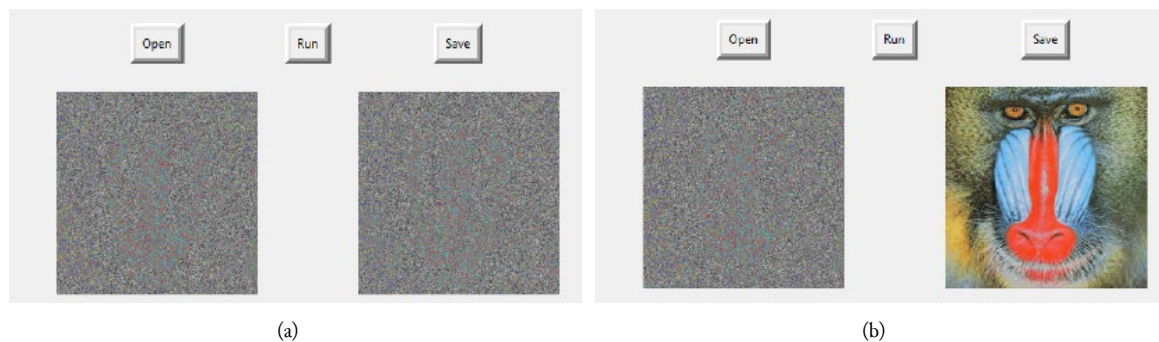


Fig. 2. (a) Decryption attempt with key change of 10^{-16} (Failure); (b) Decryption with key change of 10^{-17} (Success)

As shown in Fig. 2(a), a key change as small as 10^{-16} results in a completely unrecognizable decrypted image (MSE > 10000 , PSNR < 5 dB). Perfect decryption is only achieved when the key precision is up to 10^{-17} , as seen in Fig. 2(b) (MSE = 0, PSNR = ∞). This provides definitive quantitative and visual proof of the LC Map's extreme key sensitivity [27], surpassing the sensitivity levels reported in similar compound chaotic systems [7], [42].

3.5. Encryption Performance on Digital Images

Fig. 3 compares the histograms of the original and encrypted images to demonstrate resistance to statistical attacks.

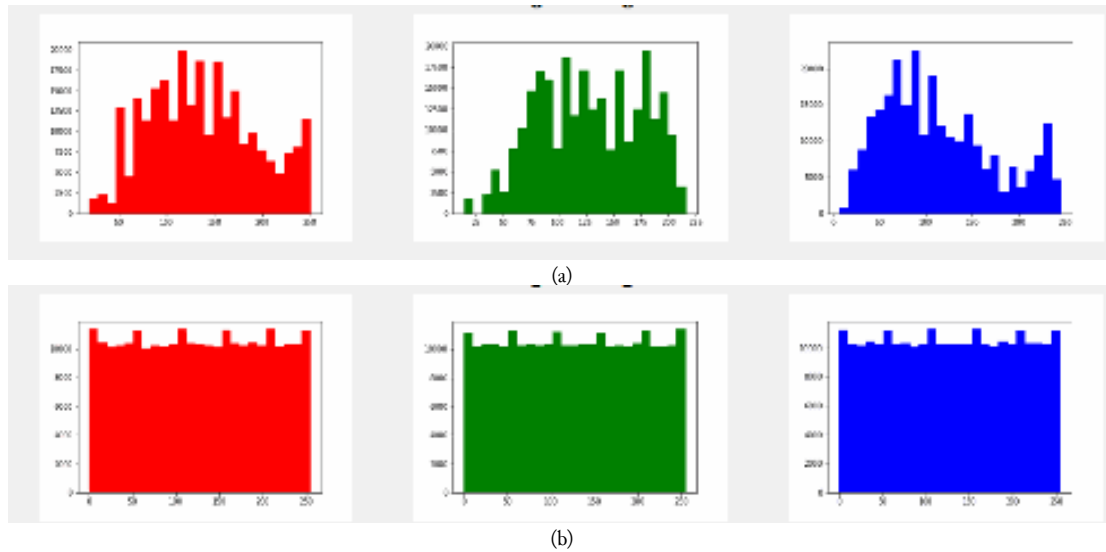


Fig. 3. Histogram of the Original Image (a) and Encrypted Image (b)

It is clear from [Fig. 3](#) that the histogram of the original image has an uneven distribution, while the histogram of the encrypted image shows a very uniform and flat distribution. This makes frequency analysis-based attacks difficult. Furthermore, after encryption, the correlation coefficient values between adjacent pixels drop drastically to near zero, proving that the LC Map algorithm successfully destroys spatial correlations

3.6. Statistical Attack Resistance Analysis on the Full Dataset

To provide extensive proof of the algorithm's robustness as requested by the reviewers, a comprehensive statistical analysis was conducted across the entire dataset of 24 test images. This section presents the detailed results for pixel correlation and information entropy, demonstrating consistent high performance across various image types and sizes.

3.6.1. Pixel Correlation Analysis

A secure encryption algorithm must effectively break the high correlation between adjacent pixels present in a plain image. The correlation coefficients were calculated for adjacent pixels in the horizontal, vertical, and diagonal directions for all 24 test images, both before (plain-image) and after (cipher-image) encryption. The results for the nine grayscale test images are presented in [Table 5](#).

Table 5. Correlation Coefficient Results for Grayscale Images

Test Data	Original Image Correlation Coefficient			Encrypted Image Correlation Coefficient		
	Horizontal	Vertical	Diagonal	Horizontal	Vertical	Diagonal
1	0.87225	0.76673	0.72990	0.00018	0.00378	-0.0011
2	0.93617	0.88382	0.84262	-0.0006	0.00209	-3.6765
3	0.97898	0.96004	0.94209	-0.0015	0.00214	0.00170
4	0.97700	0.96381	0.95852	-0.0020	-0.0005	-0.00044
5	0.99124	0.98688	0.98360	-0.0010	-0.00417	-0.0013
6	0.98501	0.97712	0.97292	-0.0019	0.00135	0.00130
7	0.96044	0.95203	0.93269	-0.0023	0.00289	-0.0035
8	0.96776	0.96539	0.94798	-0.0013	-0.0012	-0.0017
9	0.98155	0.98160	0.96920	-0.0026	-0.0057	0.00094

The analysis was extended to the 15 color images. For brevity and clarity in this journal format, the average correlation across the R, G, and B channels is presented in [Table 6](#).

Table 6. Average Correlation Coefficient Results for Color Images

Test Data	Channel	Original Image Correlation Coefficient			Encrypted Image Correlation Coefficient		
		Horizontal	Vertical	Diagonal	Horizontal	Vertical	Diagonal
10	R	0.91844	0.86029	0.84005	0.00049	0.00139	-0.0035
	G	0.88380	0.79648	0.76403	0.00155	0.00457	0.00015
	B	0.92355	0.87729	0.85478	0.00168	0.00094	-0.0015
11	R	0.98787	0.97664	0.96729	-0.00030	0.00216	-0.0020
	G	0.97595	0.95582	0.93731	-0.0016	0.00140	0.00120
	B	0.98315	0.96329	0.96329	-0.0033	-0.00160	-0.00160
12	R	0.99886	0.99841	0.99786	-0.0024	-0.00330	0.00132
	G	0.99774	0.99715	0.99589	-0.0012	-0.00280	0.00320
	B	0.99817	0.99814	0.99710	-0.0030	-0.00140	0.00208
13	R	0.98855	0.98197	0.97942	-0.0013	0.00103	0.00130
	G	0.97296	0.96487	0.95617	-0.0008	0.00073	0.00073
	B	0.98352	0.97847	0.97394	-0.0028	-0.00050	-0.00050
14	R	0.99815	0.99708	0.99625	-0.0014	0.00422	-0.0017
	G	0.99633	0.99467	0.99276	-0.0016	0.00452	-0.0012
	B	0.99712	0.99666	0.99512	-0.0029	0.00405	-0.0032
15	R	0.91844	0.86029	0.84005	0.00049	0.00139	-0.0035
	G	0.88380	0.79648	0.76403	0.00155	0.00457	0.00015
	B	0.92344	0.87729	0.85478	0.00168	0.00094	-0.0015
16	R	0.95736	0.94880	0.92784	-0.0023	0.00239	-0.0052
	G	0.96032	0.95168	0.93255	-0.0021	0.00231	-0.0036
	B	0.96554	0.95802	0.94071	-0.0003	0.00222	-0.0026
17	R	0.96440	0.96200	0.94288	-0.0014	-0.0009	-0.0019
	G	0.96807	0.96562	0.94843	-0.0015	-0.0013	-0.0013
	B	0.97290	0.97080	0.95600	-0.0022	-0.0013	-0.0021
18	R	0.97956	0.97970	0.96603	-0.0023	-0.0051	0.00102
	G	0.98169	0.98170	0.96941	-0.0027	-0.0055	0.00097
	B	0.98456	0.98457	0.97412	-0.0025	-0.0050	0.00121
19	R	0.96628	0.95570	0.93483	-0.0039	-0.0107	0.00153
	G	0.95646	0.94164	0.91683	0.00180	-0.0086	0.00645
	B	0.94978	0.92715	0.89866	-0.0051	-0.0056	0.00508
20	R	0.89300	0.89954	0.83252	0.00049	-0.0028	-0.0030
	G	0.88858	0.89721	0.82634	0.00068	-0.0016	-0.0035
	B	0.90833	0.91448	0.85792	-0.0017	7.04370	-0.0032
21	R	0.95985	0.96243	0.93508	-0.0033	-0.0011	0.00201
	G	0.95768	0.96053	0.93051	-0.0037	0.00090	0.00044
	B	0.95862	0.96189	0.93599	-0.0034	0.00094	0.00078
22	R	0.89740	0.90212	0.85936	-0.0016	-0.0004	-0.0007
	G	0.83980	0.84770	0.78305	-0.0020	0.00097	-0.0002
	B	0.68500	0.69975	0.59439	-0.0034	0.00171	-0.0030
23	R	0.98967	0.98860	0.98130	-0.0015	-0.0038	-0.0034
	G	0.98784	0.98653	0.97820	-0.0023	-0.0035	-0.0022
	B	0.98618	0.98487	0.97512	-0.0025	-0.0040	-0.0022
24	R	0.95659	0.95371	0.93004	-0.0026	0.00107	0.00359
	G	0.94843	0.94898	0.91253	-0.0028	0.00118	0.00359
	B	0.94769	0.95128	0.91047	-0.0023	0.00066	0.00371

The correlation analysis results presented in Table 5 and Table 6 are definitive. For all 24 test images, the correlation coefficients in the plain images are consistently high (approaching 1). After encryption with the LC Map algorithm, these correlation values drop drastically to values near zero. This demonstrates that the algorithm successfully destroys the spatial correlation, proving its high resistance to statistical analysis attacks

4. Conclusion

This research has successfully developed and validated the LC Map as a foundation for a robust and efficient digital image encryption algorithm. The comprehensive analysis has proven that the LC Map offers significant advantages over conventional single-dimensional chaotic maps. The main contributions of this research have been confirmed through a series of tests. First, the LC Map exhibits strong chaotic behavior, as evidenced by dense bifurcation diagrams, consistently positive Lyapunov Exponent values, and a 100% pass rate on all 16 NIST statistical tests under optimal parameters. Second, from a security perspective, the LC Map-based encryption algorithm demonstrates superior resistance to various types of attacks. This is evidenced by a computationally infeasible key space (7.2×10^{958}), extreme key sensitivity (10–17), and near-ideal NPCR and UACI values. Functionally, the algorithm has been proven effective and lossless, with the decryption process perfectly restoring the original image ($MSE = 0$ and $PSNR = \infty$). These findings, combined with better computational efficiency compared to sequential combination methods, position the LC Map as a significant contribution to the field of chaos-based cryptography. Future research could focus on exploring hardware implementations of the LC Map and extending its application to other types of multimedia data.

Declarations

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