Type-2 Fuzzy ANP and TOPSIS methods based on trapezoid Fuzzy number with a new metric

Yeni Kustiyahningsih [a,1,*], Eza Rahmanita [a,2], Bain Khusnul Khotimah [a,3], Jaka Purnama [b,4]

[a] University of Trunojoyo Madura, Jl. Raya Telang PO BOX 2, Bangkalan and 69162, Indonesia
[b] University of 17 Agustus 1945, Jl. Semolowaru 45, Surabaya and 60118, Indonesia
[1] ykustiyahningsih@trunojoyo.ac.id; [2] eza.rahmanita@trunojoyo.ac.id; [3] bain@trunojoyo.ac.id; [4] jakapurnama@untag-sby.ac.id
* corresponding author

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ABSTRACT

Modeling and linguistic representation in the form Interval Type-2 Fuzzy have better accuracy than Type-1 Fuzzy. The type-2 fuzzy set involves more uncertainty than the type-1 fuzzy set. The degree of fuzzy membership is used to explain uncertainty and ambiguity in the real world. This study presents the type-2 Fuzzy Analytic Network Process (ANP) method to determine the weight of each attribute based on the level of interest and the extension method of type-2 Fuzzy TOPSIS to handle problems based on the value of the fuzzy type-2 attribute. Decision-making is based on the assessment of several experts called Multi-Criteria Group Decision Making (MCGDM), using type-2 Fuzzy geometric mean aggregation function. The membership function in this research is type-2 fuzzy based on the trapezoid. The contribution is a hybrid method Type-2 Fuzzy TOPSIS with Fuzzy Type-2 ANP group-based with new metric intervals on fuzzy type-2 for decision making. The results are a hybrid type-2 FANP and FTOPSIS decision-making model to support the best decision-making. Based on a comparison of the accuracy of trapezoid model 1, model 2, and model 3, the best accuracy result is model 3, which is 84%. The research benefits by presenting a hybrid Type-2 Fuzzy TOPSIS and ANP method that improves decision-making accuracy and better handling uncertainty and ambiguity than Type-1 Fuzzy systems.

1. Introduction

The purpose of a group-based decision support system or called multi-attribute group decision making (MAGDM) is to determine the best decision or alternative from several alternatives given based on several experts or assessors. Determination of alternatives by selecting and evaluating the best alternatives based on many specified attributes or criteria. Several previous studies have determined the selected alternative based on many criteria using the fuzzy type-1 method [1]–[5]. The use of fuzzy type-1 method to determine membership function in determining a selection of new student registrations. The fuzzy method is also a hybrid with AHP and COPRAS [6]. The fuzzy method is used to assess preparation in e-government from a CiRM perspective [7]. Group-based decision support to determine the best e-learning content using the AHP fuzzy function [8].

Research on fuzzy and Analytic Hierarchy Process (AHP) for IoT evaluation [9]. Fuzzy to determine membership function supplier by integrating with TOPSIS. The fuzzy method integrated with AHP is used to measure the quality of system management for blended learning using FAHP [10]. TOPSIS fuzzy method is used to improve network selection in a multi-access wireless environment [11]. Fuzzy

Analytic Network Process (FANP) is a combination of two methods, namely Fuzzy and Analytic Network Process (ANP) [12]. These two methods are combined because the fuzzy method can cover uncertain data and help unclear decision makers. While the Analytic Network Process (ANP) method can determine the existence of dependencies between criteria and between sub-criteria. The Fuzzy Analytic Network Process (FANP) considers the dependency relationship between criteria and sub-criteria. The FANP method is used to find the priority weights of all the criteria and sub-criteria that have been determined and as a ranking process for the weight of the criteria for each alternative data [12], [13].

TOPSIS has simple computation, ANP, and TOPSIS methods used in decision-making for evaluating and selecting software quality [14]. Group decisions on FMCGDM can be made by generating decision-making preferences for each given preference. In group decisions, there is a better level of consensus, but linguistic preferences will overlap. Overlapping on group decisions can use type-2 Fuzzy. Type-2 Fuzzy is the development of type-1 Fuzzy [15]. Modeling generated at Interval Type-2 Fuzzy is more accurate, and the rating performance is better. The results of the linguistic representation on the type-2 interval are more effective and flexible than ordinary fuzzy [16]–[18]. Type-2 Fuzzy Interval has clearer linguistic modeling, so it can increase accuracy [18]–[20].

Fuzzy type-2 interval can be used to improve accuracy by optimizing the number and position of the fuzzy set [21], [22]. Therefore, in this study, the fuzzy type-2 trapezoidal fuzzy number method is used for the decision-making model of the selection group or the selection of SMEs. Based on this approach, this research was developed by doing a hybrid Interval type-2 Fuzzy Analytic Network Process (IT2-FANP) with interval type-2 Fuzzy Technique for Order Preference of Similarity Ideal Solution (IT2 FTOPSIS) with modification of interval points on the function fuzzy trapezium for decision making. This research produces a decision-making model and a new algorithm based on Fuzzy Interval type-2 with linguistic preferences that are more flexible and have a high level of accuracy.

2. Method

2.1. Membership Function type-2 Trapezoid

Zadeh introduced fuzzy logic, which is used to make decisions based on unclear, incomplete, and approximate information. The trapezoidal fuzzy number can be defined as.

$$A^U = (S_1^u, S_2^{mu}, S_3^{mu}, S_4^u, \mu_A^u), S_1^u < S_2^{mu} < S_3^{mu} < S_4^u \quad (1)$$

This function approach uses a trapezoidal curve or type-2, as shown in Fig. 1.

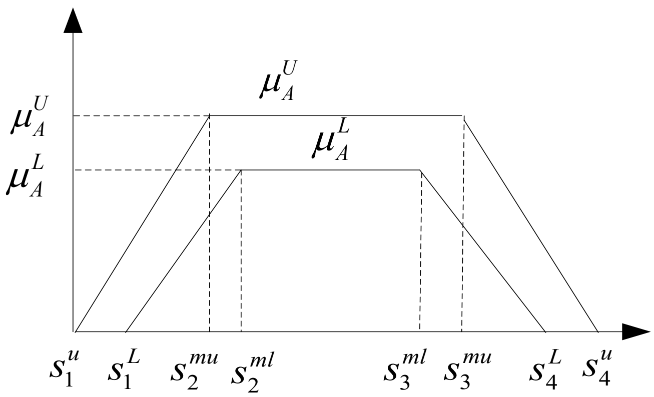

Fig. 1. Interval type-2 Trapezoid Fuzzy Number [21]

According to Fig. 1, the membership function is defined as follows [23].

$$\mu_A^L(x) = \begin{Bmatrix} \frac{x-S_1^l}{S_2^{ml}-S_1^l}; S_1^l \le x < S_2^{ml} \\ \frac{S_4^{ml}-x}{S_4^l-S_3^{ml}}; S_3^{ml} \le x < S_4^l \\ 0; \quad x < S_1^l or\ x \ \ge S_4^l \\ 1; \quad S_2^{ml} \le x < S_3^{ml} \end{Bmatrix} \tag{2}$$

With $A^L = S_1^l, S_2^{ml}, S_3^{ml}, S_4^{ml}), S_1^l < S_2^{ml}\ S_3^{ml} S_4^{ml}$

$$\mu_A^U(x) = \begin{Bmatrix} \frac{x-S_1^u}{S_2^{mu}-S_1^u}; S_1^u \le x < S_2^{mu} \\ \frac{(S_4^{mu}-x)}{S_4^u-S_3^{mu}};\ S_3^{mu} \le x < S_4^u \\ 0; \quad x < S_1^u or\ x \ge\ S_4^u \\ S_2^{mu} \quad \le x < S_3^{mu} \end{Bmatrix} \tag{3}$$

With $A^U = S_1^u, S_2^{mu}, S_3^{mu}, S_4^u), S_1^u < S_2^{mu} < S_3^u < S_4^u\ , 0 \le \mu_A^L \le \mu_A^U \le 1, A^L \subset A^U$

Arithmetic operations for Interval type-2 Trapezoid Fuzzy Number numbers can be seen from the following equation [24]:

$$P = [P^L, P^U] = [(p_1^l, p_2^{ml}, p_3^{ml}, p_4^l), (p_1^u, p_2^{mu}, p_3^{mu}, p_4^u)]$$

$$S = [S^L, S^U] = [(S_1^l, S_2^{ml}, S_3^{ml}, S_4^l), (S_1^u, S_2^{mu}, S_3^{mu}, S_4^u)] \tag{4}$$

With $p_1^u < p_2^{mu} < p_3^{mu} < p_4^u, S_1^u < S_2^{mu} < S_3^{mu} < S_4^u$

The arithmetic operations of two type-2 Trapezoid Fuzzy Numbers are:

- The operation of adding two type-2 Trapezoid Fuzzy Number P, S

$$P + S = [(p_1^l, p_2^{ml}, p_3^{ml}, p_4^l), (p_1^u, p_2^{mu}, p_3^{mu}, p_4^u)] + [(S_1^l, S_2^{ml}, S_3^{ml}, S_4^l), (S_1^u, S_2^{mu}, S_3^{mu}, S_4^u)]$$

$$= \left[\left(p_1^l + S_1^i, p_2^{ml} + S_2^{ml}, p_3^{ml} + S_3^{ml}, p_4^1 + S_4^1\right)\right), (p_1^u + S_1^u, p_2^{mu} + S_2^{mu}, p_3^{mu} + S_3^{mu}, p_4^u + S_4^u))] \tag{5}$$

- The operation of adding two Interval type-2 Trapezoid Fuzzy Number P, S

$$P - S = [(p_1^l, p_2^{ml}, p_3^{ml}, p_4^l), (p_1^u, p_2^{mu}, p_3^{mu}, p_4^u)] - [(S_1^l, S_2^{ml}, S_3^{ml}, S_4^l), (S_1^u, S_2^{mu}, S_3^{mu}, S_4^u)]$$

$$= \left[\left(p_1^l - S_4^i, p_2^{ml} - S_3^{ml}, p_3^{ml} - S_2^{ml}, p_4^1 - S_1^l\right)\right), (p_1^u - S_4^u, p_2^{mu} - S_3^{mu}, p_3^{mu} - S_2^{mu}, p_4^u - S_1^u)] \tag{6}$$

- Multiplication operation of two Interval type-2 Trapezoid Fuzzy Number P, S

$$P.S = [(p_1^l, p_2^{ml}, p_3^{ml}, p_4^l), (p_1^u, p_2^{mu}, p_3^{mu}, p_4^u)].[(S_1^l, S_2^{ml}, S_3^{ml}, S_4^l), (S_1^u, S_2^{mu}, S_3^{mu}, S_4^u)]$$

$$= [(p_1^l.S_1^l, p_3^{ml}.S_3^{ml}, p_4^l.S_4^l), (p_1^u.S_1^u, p_2^{mu}.S_2^{mu}, p_3^{mu}, p_4^u.S_4^u)] \tag{7}$$

- The operation of dividing two Interval type-2 Trapezoid Fuzzy Number P, S

$$\frac{P}{S} = \frac{[(p_1^i, p_2^{ml}, p_3^{ml}, p_4^1), (p_1^u, p_2^{mu}, p_3^{mu}, p_4^u)]}{[(S_1^l, S_2^{ml}, S_3^{ml}, S_4^l), (S_1^u, S_2^{mu}, S_3^{mu}, S_4^u)]}$$

$$= \left[\left(\frac{p_1^l}{S_4^l}, \frac{p_2^{ml}}{S_3^{ml}}, \frac{p_3^{ml}}{S_2^{ml}}, \frac{p_4^l}{S_1^l}\right), \left(\frac{p_1^u}{S_4^u}, \frac{p_2^{mu}}{S_3^{mu}}, \frac{p_3^{mu}}{S_2^{mu}}, \frac{p_4^u}{S_1^u}\right)\right] \tag{8}$$

- Multiplication operation type-2 Trapezoid Fuzzy Number P and constant number

$$\lambda P = \lambda. [(p_1^l, p_2^{ml}, p_3^{ml}, p_4^l), (p_1^u, p_2^{mu}, p_3^{mu}, p_4^u)]$$

$$[(\lambda.p_1^{l}, \lambda.p_2^{ml}, \lambda.p_3^{ml}, \lambda.p_4^{ml}, (\lambda.p_1^{u}, \lambda.p_2^{mu}, \lambda.p_3^{mu}, \lambda.p_4^{mu} \quad (9)$$

2.2. Hybrid type-2 FANP and Extension TOPSIS methods

The research method is shown in Fig. 2. Hybrid type-2 Fuzzy ANP and TOPSIS. This method is divided into 2 stages; the first stage is type-2 Fuzzy ANP. This method begins with determining criteria and sub-criteria indicators first. Then determine trapezoid-based fuzzy type-2 with scale, pairwise comparison, geometric mean, calculate the smallest CR value, calculate synthetic extent value, and get criterion weight in the form of a type-2 fuzzy number. This interval weight will be included in the next method. If you want to get weights in non-fuzzy or crisp form, then proceed to the next step. After that was completed, the process continued with trapezoid-based FTOPSIS [15], [25].

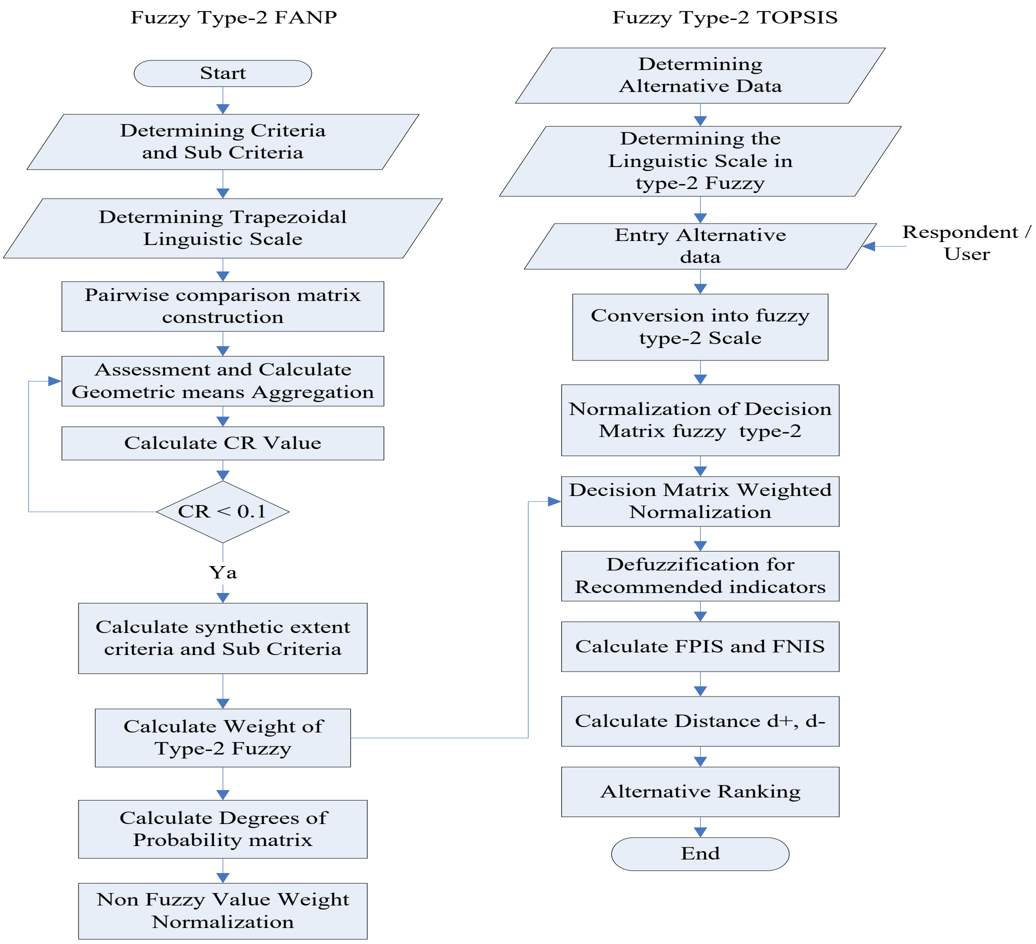

Fig. 2. Hybrid type-2 FANP and TOPSIS based on Trapezoid fuzzy number

3. Results and Discussion

3.1. Hybrid type-2 FANP and FTOPSIS Model with Trapezoid function

The steps for the hybrid method of interval type-2 Fuzzy ANP and interval type-2 FTOPSIS are Constructing a multi-criteria decision-making model using the type-2 FANP method on the Trapezoid Fuzzy Number function with different points according to Fig. 3. Steps at this stage are as follows [26],[27]:

- Step 1. The comparison matrix construction between criteria is denoted by matrix D.

$$D = \begin{bmatrix} & k_1 & k_2 & \cdots & k_n \\ k_1 & 1 & d_{12} & \cdots & d_{1n} \\ k_2 & d_{21} & 1 & \cdots & d_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ k_n & d_{n1} & d_{n2} & \cdots & 1 \end{bmatrix} \quad (10$$

With n = Number of criteria, k = Criteria, d= matrix element D

- Step 2. The conversion of the pairwise comparison D matrix into a type-2 trapezoidal fuzzy number interval is shown in Fig. 1.

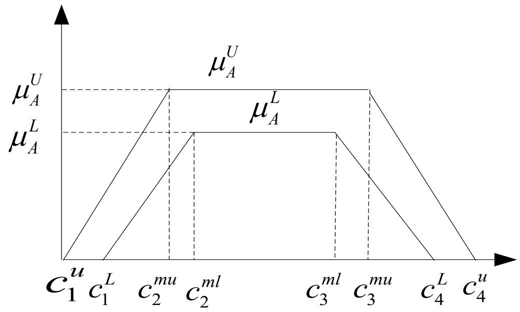

Fig. 3. Interval type-2 Trapezoid fuzzy number

Based on Fig. 3., matrix C can be expressed as follows:

$$C = \begin{bmatrix} 1 & c_{12} & c_{12} & \cdots & c_{1n} \\ c_{21} & 1 & c_{23} & \cdots & c_{2n} \\ c_{31} & c_{32} & 1 & \cdots & c_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & c_{n3} & \cdots & 1 \end{bmatrix} \; with\; i,j = 1,2,\ldots n \quad (11)$$

$$C_{ij} = \left(c_{1ij}^{l}, c_{2ij}^{ml}, c_{3ij}^{ml}, c_{4ij}^{l}, c_{1ij}^{u}, c_{2ij}^{mu}, c_{3ij}^{mu}, c_{4ij}^{u}\; ;\; \mu_A^L, \mu_A^U\right), c_{ij}^{-1}$$

$$= \left(\frac{1}{c_{4ij}^{u}}, \frac{1}{c_{4ij}^{l}}, \frac{1}{c_{3ij}^{mu}}, \frac{1}{c_{3ij}^{ml}}, \frac{1}{c_{2ij}^{mu}}, \frac{1}{c_{2ij}^{ml}}, \frac{1}{c_{1ij}^{u}}, \frac{1}{c_{1ij}^{l}}; \mu_A^L, \mu_A^U\right), \quad (12)$$

C = Pairwise matrix, μ_A^l = Membership Lower limit, μ_A^u = Membership upper limit, c_{ij} = Criteria, c_{ij}^{-1} = Reciprocal row

- Step 3. Calculate the fuzzy synthesis value for the i-th object, which is defined as follows $\mathrm{U} = \{\mathrm{u}_1, \mathrm{u}_1, \ldots, \mathrm{u}_\mathrm{n}\}$ set of objects and $\mathrm{U} = \{\mathrm{u}_1, \mathrm{u}_1, \ldots, \mathrm{u}_\mathrm{n}\}$ goal set. Each object is taken, and an expansion analysis is carried out for each purposeg_i. Therefore, the value of the analysis of the expansion of m for each object is obtained $\mathrm{K}_{\mathrm{gi}}^1, \mathrm{K}_{\mathrm{gi}}^2, \ldots, \mathrm{K}_{\mathrm{gi}}^\mathrm{m}; i = 1,2, \ldots, n$ with $\mathrm{K}_{\mathrm{gi}}^\mathrm{j}(\mathrm{j} = 1,2, \ldots, \mathrm{m})$

$$S_i = \sum_j^m K_{gi}^j \otimes \left[\sum_{j=1}^n \sum_{j=1}^m K_{gi}^j\right]^{-1} \quad (13)$$

To obtain M_{gi}^j , operations perform additional fuzzy m operations with a certain matrix

$$\sum_{j=1}^m K_{gi}^j = \left[\sum_{j=1}^m p_{ij}^l, \sum_{j=1}^m p_{ij}^u, \sum_{j=1}^m p_{2j}^{ml}, \sum_{j=1}^m p_{2j}^{mu}, \sum_{j=1}^m p_{3j}^{ml}, \sum_{j=1}^m p_{3j}^{mu}, \sum_{j=1}^m p_{4j}^l, \sum_{j=1}^m p_{4j}^u\right] \quad (14)$$

To obtain $\left[\sum_{j=1}^n \sum_{j=1}^m K_{gi}^j\right]$ a fuzzy operation from the value $K_{gi}^j(j = 1,2,3 \ldots m)$ as below:

$$\sum_{i=1}^n \sum_j^m = 1\, K_{gi}^j =$$
$$\left(\sum_{i=1}^n p^u \;\sum_{4i}^n{}_{i=1} p^l, \sum_{4i}^n{}_{i=1} p^{mu}, \sum_{3i}^n{}_{i=1} p^{ml}, \sum_{3i}^n{}_{i=1} p^{ml}, \sum_{2i}^n{}_{i=1} p^{mu}, \sum_{i=1}^n{}_{1i} p^l, \sum_{i=1}^n{}_{1i} p^u\right) \quad (15)$$

The inverse of the vector determinant :

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m}K_{gi}^{j}\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}p_{4i}^{u}}, \frac{1}{\sum_{i=1}^{n}p_{4i}^{l}}, \frac{1}{\sum_{i=1}^{n}p_{3i}^{mu}}, \frac{1}{\sum_{i=1}^{n}p_{3i}^{ml}}, \frac{1}{\sum_{i=1}^{n}p_{2i}^{ml}}, \frac{1}{\sum_{i=1}^{n}p_{2i}^{mu}}, \frac{1}{\sum_{i=1}^{n}p_{1i}^{l}}, \frac{1}{\sum_{i=1}^{n}p_{1i}^{mu}}\right) \quad (16)$$

- Step 4. Calculate Possible Degrees of M

$$K2(p_1^{u1}, p_1^{l1}, p_2^{mu1}, p_2^{ml1}, p_3^{ml1}, p_3^{mu1}, p_4^{l1}, p_4^{u1})^3 = K_1(p_1^{u2}, p_1^{l2}, p_2^{mu2}, p_2^{ml2}, p_3^{ml2}, p_3^{mu2}, p_4^{l2}, p_4^{u2}) \quad (17)$$

Defined as follows:

$$V(K_2 \geq K_1) = \sup\,[\min\,(\mu_{K1}(x), \mu_{K1}(y)] \quad (18)$$

It can be expressed equivalently as follows:

$$V(K_2 \geq K_1) = hgt(K_2 \cap K_1) = \mu_{M2}(d) = \begin{cases} 1, & if\ m_2 \geq m_2 \\ 0, & if\ l_2 \geq u_2 \\ \frac{i_1 - u_2}{(m_2 - u_2) - (m_1 - l_2)}, & Otherwise \end{cases} \quad (19)$$

- Step 5. Calculating probability degree for convex of fuzzy numbers greater than k in convex of fuzzy defined as:

$K_i = (i = 1, 2, \dots, k)$ then the vector value can be defined as follows:

$$V(K \geq K_1, K_2, \dots, K_K)$$

$$= V[K \geq K_1) and\ (K \geq K_2) \dots and (K \geq K_k)]$$

$$= \min V(K \geq K_i), i = 1,2, \dots, k \quad (20)$$

It is assumed that

$$d'(A_i) = \min V(S_i \geq S_k) k = 1,2, \dots, n; k \neq i \quad (21)$$

Vector weight using $W' = (d'(A_1), d'(A_1), \dots, d'(A_n))^T$
With $A_i = 1,2, \dots, n$ is the sum of n decision elements

- Step 6. Normalization of weights: $W = (d(A_1), d(A_2), \dots, d(A_n))^T$ where W is the weight of a non-fuzzy number.
- Step 7. Construct an MCDM model with type-2 FTOPSIS on a Trapezoid function with different midpoints and the same degree of membership according to Fig. 4. The steps at this stage are as follows:
  - Constructing the alternative decision matrix against the criteria according to Fig. 4. The decision matrix for each alternative against the criteria is matrix V as follows [28].

$$S = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} \\ S_{21} & S_{22} & \cdots & S_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} \end{bmatrix} \quad (22)$$

With $v_{ij} = [(S_{ij1}^{l} S_{ij2}^{ml}, S_{ij3}^{ml}, S_{ij4}^{l}, S_{ij}^{l}), (S_{ij1}^{u}, S_{ij2}^{mu}, S_{ij3}^{mu}, S_{ij4}^{u}, S_{ij}^{u})]$

V = Comparison matrix between alternatives with criteria
v_{ij} = Matrix elements V with i-th alternative and j-th criteria

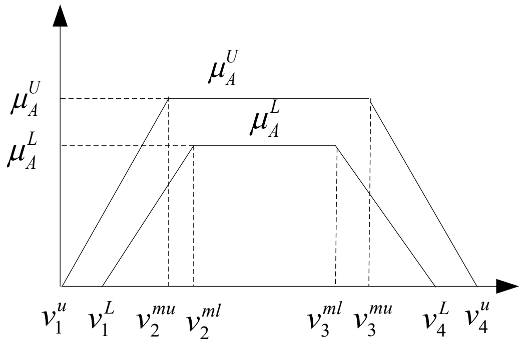

Fig. 4. Type-2 Trapezoidal fuzzy Number model 1

Fig. 4. is a form of decision-making model Interval type-2 Trapezoid Fuzzy Number with different middle points $(v_{2ij}^{ml} \neq v_{2ij}^{mu}, v_{3ij}^{ml} \neq v_{3ij}^{mu})$ and $(\mu_A^1 < \mu_A^u)$.

- Constructing a normalized decision matrix based on the V matrix, which is denoted by S, can be expressed as follows:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2n} \\ S_{31} & S_{32} & S_{33} & \cdots & S_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & S_{n3} & \cdots & S_{nm} \end{bmatrix} \quad (17)$$

$$S_{ij} = \left[\frac{v_{1ij}^u}{c_j^+}, \frac{v_{2ij}^{mu}}{c_j^+}, \frac{v_{3ij}^{mu}}{c_j^+}, \frac{v_{4ij}^u}{c_j^+}, \frac{v_{1ij}^l}{c_j^+}, \frac{v_{2ij}^{ml}}{c_j^+}, \frac{v_{3ij}^{ml}}{c_j^+}, \frac{v_{4ij}^l}{c_j^+}\right] j = 1,2,\dots,n. or$$

$$S_{ij} = \left[\frac{a_j^-}{v_{4ij}^u}, \frac{a_j^-}{v_{3ij}^{mu}}, \frac{a_j^-}{v_{2ij}^{mu}}, \frac{a_j^-}{v_{1ij}^u}, \frac{a_j^-}{v_{4ij}^l}, \frac{a_j^-}{v_{3ij}^{ml}}, \frac{a_j^-}{v_{2ij}^{ml}}, \frac{a_j^-}{v_{1ij}^l}\right] j = 1,2,\dots,n$$

With

$c_j^+ = \max(v_{ij}, v_{2j}, \dots, v_{nj}),$

$a_j^- = \min(v_{ij}, v_{2j}, \dots, v_{nj}), j = 1, 2, 3, \dots, n,$

$v_{1ij}^u < v_{1ij}^l < v_{2ij}^{mu} < v_{2ij}^{ml} < v_{3ij}^{mu} < v_{2ij}^{ml} < v_{3ij}^l < v_{3ij}^u$

- Constructing a weight normalized denoted by B can be expressed as follows:

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \quad (20)$$

$b_{ij} = w_j.S_{ij}$

$= (S_{1ij}^u.w_j, S_{2ij}^{mu}.w_j, S_{3ij}^{mu}.w_j, S_{4ij}^u.w_j, S_{1ij}^l.w_j, S_{2ij}^{ml}.w_j, S_{3ij}^{ml}.w_j, S_{4ij}^l.w_j)$

with $i = 1, 2, 3, \dots, n$; and $j = 1, 2, 3, \dots, n$

- The positive ideal is k_j^+ which is $k_j^+ = \max(b_{1j}, b_{2j}, \dots, b_{nj}$ with is $j = 1, 2, 3, \dots, n$. The negative ideal is $\tilde{v}_{\bar{j}}$ which $k_j^- = m\ in\ (b_{1j}, b_{2j}, \dots, b_{nj})$, with $j = 1, 2, 3, \dots, n$.

The distance is denoted d_i^+ which is stated as follows:

$$d_{i1}^{+} = \sum_{j=1}^{n} \sqrt{\frac{1}{n}\left[(b_{1ij}^{u} - k_j^{+})^2 + (b_{2ij}^{mu} - k_j^{+})^2 + (b_{3ij}^{mu} - k_j^{+})^2 + (b_{4ij}^{u} - k_j^{+})^2\right]} \quad (21)$$

$$d_{i2}^{+} = \sum_{j=1}^{n} \sqrt{\frac{1}{n}\left[(b_{1ij}^{i} - k_j^{+})^2 + (b_{2ij}^{ml} - k_j^{+})^2 + (b_{3ij}^{ml} - k_j^{+})^2 + (b_{4ij}^{l} - k_j^{+})^2\right]} \quad (22)$$

The distance of each alternative from the negative ideal solution is denoted by d_i^- which is stated as follows:

$$d_{i1}^{-} = \sum_{j=1}^{n} \sqrt{\frac{1}{n}\left[(b_{1ij}^{u} - k_j^{-})^2 + (b_{2ij}^{mu} - k_j^{-})^2 + (b_{3ij}^{mu} - k_j^{-})^2 + (b_{4ij}^{u} - k_j^{-})^2\right]} \quad (23)$$

$$d_{i2}^{-} = \sum_{j=1}^{n} \sqrt{\frac{1}{n}\left[(b_{1ij}^{l} - k_j^{-})^2 + (b_{2ij}^{ml} - k_j^{-})^2 + (b_{3ij}^{ml} - k_j^{-})^2 + (b_{4ij}^{l} - k_j^{-})^2\right]} \quad (24)$$

with n = Trapezoid interval point

- Calculates relative proximity can be denoted by which is stated as follows:

$$RC_i \quad RC_{i1} = \frac{d_{i1}^{-}}{d_{i1}^{+} + d_{i1}^{-}} \; and \; RC_{i2} = \frac{d_{i1}^{-}}{d_{i2}^{+} + d_{i2}^{-}}$$

$$RC_i = \frac{RC_{i1} + RC_{i2}}{2} \quad (25)$$

Alternate ranking, Alternative by value RC_i

3.2. Type-2 Fuzzy Trapezoid number with new metric

Type-2 Fuzzy on Trapezoid function with different midpoints, different degrees of membership, and new metric. The steps Hybrid type-2 FANP and FTOPSIS are the same in sub-chapter 3.1.; only the size of the linguistic scale is different. The construction of the decision-making model with the new metric is shown in Fig. 5. and Fig. 6:

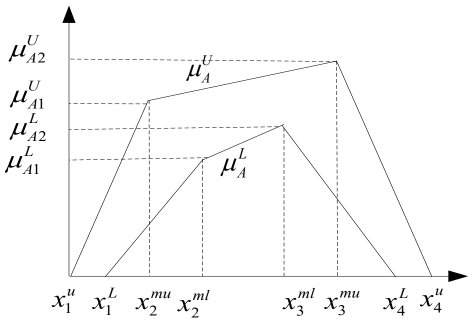

Fig. 5. Trapezoid function with different midpoints and different degrees of membership model-2

Fig. 6. is a form of decision-making model using a Fuzzy Trapezoid with different middle points $(x_{2ij}^{ml} \neq x_{2ij}^{mu}), (x_{3ij}^{ml} \neq x_{3ij}^{mu})$ and different degrees of membership $(\mu_A^{i} \leq \mu_A^{u}, \mu_{A1}^{1} \leq \mu_{A2}^{l}, \mu_{A1}^{u} \leq \mu_{A2}^{u}$

Fig. 6. is a form Trapezoid Fuzzy Number model with different points $(x_{2ij}^{ml} \neq x_{2ij}^{mu}), (x_{3ij}^{ml} \neq x_{3ij}^{mu})$ and different degrees of membership $(\mu_A^{L} \leq \mu_A^{U}, \mu_{A2}^{L} \leq \mu_{A1}^{L}, \mu_{A2}^{U} \geq \mu_{A1}^{U}$.

Based on Fig. 5 and Fig. 6, matrix X can be expressed as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & \cdots & x_{nm} \end{bmatrix} \quad (26)$$

With

$$x_{ij} = [(x^l_{ij1}, x^{ml}_{ij2}, x^{ml}_{ij3}, x^l_{ij4}, \mu^l_{A1}, \mu^l_{A2}), (x^u_{ij1}, x^{mu}_{ij2}, x^{mu}_{ij3}, x^u_{ij4}, \mu^u_{A1}, \mu^u_{A2})] \quad (27)$$

$$i, j = 1, 2, 3, \ldots, n$$

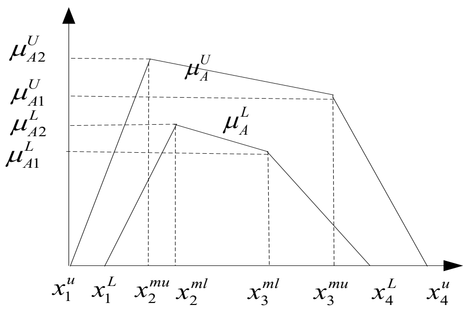

Fig. 6. Trapezoid function with different midpoints and degree of membership model-3

3.3. Example calculation for Implementation of hybrid method

The steps for implementation of Hybrid Model for selection of Madura batik SMEs with interval type-2 FANP and TOPSIS with Trapezoid fuzzy number.

- Determine the number of criteria and alternatives for ranking. The criteria for SMEs are Batik Motif Variations (C1), Shop Ownership (C2), and marketplace (C3).
- Determining the scale of type-2 fuzzy using a trapezoid can be seen in Table 1.

Table 1. Definition Linguistic Scales Criteria of interval Type-2 Fuzzy

Numeric	Type-2 Fuzzy Scale	Definition
	[(1,1,1,1) (1,1,1,1)]	The Same Criteria
1	[(0.1, 0.5,1.3, 1.7;1,1) (0.3,0.7,1,1.5;0.9,0.9)]	Equally Important
3	[(1.5,2,2.7,3.2;1,1) (1.7,2.2, 2.5, 3;0.9,0.9)]	A Little More Important
5	[(3, 3.5, 4.2, 4.7;1,1) (3.2, 3.7, 4, 4.5;0.9,0.9)]	More important
7	[(4.5,5, 6, 6.5;1,1) (4.7, 5.2, 5.6, 6.3;0.9,0.9)]	Very More Important
9	[(6.3, 6.7, 7.5, 8;1,1) (6.5,7,7.3,7.7;0.9,0.9)]	The most important

- Conducted questionnaires to 3 experts, namely SME service, researchers and practitioners. The results of the expert assessment can be seen in Table 2, Table 3, and Table 4.

Table 2. Expert 1

	C1								C2	C3				
	v^u_1	v^{mu}_2	v^{mu}_3	v^u_4	v^L_1	v^{ml}_2	v^{ml}_3	v^L_4	...	v^u_1	v^{mu}_2	v^{mu}_3	v^u_4	v^L_1
C1	1	1	1	1	1	1	1	1	⋯	0.31	0.37	0.5	0.67	0.33
C2	1.5	2	2.7	3.2	1.7	2.2	2.5	3	⋯	0.31	0.37	0.5	0.67	0.33
C3	1.5	2	2.7	3.2	1.7	2.2	2.5	3	⋯	1	1	1	1	1

Table 3. Expert 2

	C1								C2	C3				
	v^u_1	v^{mu}_2	v^{mu}_3	v^u_4	v^L_1	v^{ml}_2	v^{ml}_3	v^L_4	...	v^u_1	v^{mu}_2	v^{mu}_3	v^u_4	v^L_1
C1	1	1	1	1	1	1	1	1	⋯	0.31	0.37	0.5	0.6	0.33
C2	1.5	2	2.7	3.2	1.7	2.2	2.5	3	⋯	0.31	0.37	0.5	0.6	0.33
C3	1.5	2	2.7	3.2	1.7	2.2	2.5	3	⋯	1	1	1	1	1

Table 4. Geometric means aggregation trapezoid

	C1								C2			C3		
	v_1^u	v_2^{mu}	v_3^{mu}	v_4^u	v_1^L	v_2^{ml}	v_3^{ml}	v_4^L	…	v_1^u	v_2^{mu}	v_3^{mu}	v_4^u	v_1^L
C1	1	1	1	1	1	1	1	1	⋯	0.31	0.37	0.5	0.6	0.3
C2	1.23	1.42	1.66	1.81	1.31	1.49	1.59	1.75	⋯	0.31	0.37	0.5	0.6	0.3
C3	1.23	1.42	1.66	1.81	1.31	1.49	1.59	1.75	⋯	1	1	1	1	1

Calculate Geometric Means based on trapezoid, which is shown in Table 4, and calculate synthetic extent criteria as in Table 5, proceed with calculating weighted normalization as in Table 6.

Table 5. Results of synthetic Extend Criteria

	$\sum v_1^u$	$\sum v_2^{mu}$	$\sum v_3^{mu}$	$\sum v_4^u$	$\sum v_1^L$	$\sum v_2^{ml}$	$\sum v_3^{ml}$	$\sum v_4^L$
C1	1.63	1.75	2.01	2.34	1.67	1.81	1.92	2.18
C2	2.81	3.36	4.18	4.83	3.03	3.59	3.94	4.56
C3	3.99	4.97	6.35	7.33	4.38	5.37	5.95	6.93
Total	8.43	10.08	12.53	14.50	9.08	10.76	11.81	13.68
	$1/\sum v_1^u$	$1/\sum v_2^{mu}$	$1/\sum v_3^{mu}$	$1/\sum v_4^u$	$1/\sum v_1^L$	$1/\sum v_2^{ml}$	$1/\sum v_3^{ml}$	$1/\sum v_4^L$
	0.12	0.10	0.08	0.07	0.11	0.09	0.08	0.07

Table 6. Weight of interval type-2 Trapezoid

	v_1^u	v_2^{mu}	v_3^{mu}	v_4^u	v_1^L	v_2^{ml}	v_3^{ml}	v_4^L
C1	0.12	0.15	0.19	0.26	0.12	0.14	0.19	0.26
C2	0.21	0.28	0.39	0.53	0.21	0.29	0.39	0.54
C3	0.29	0.42	0.59	0.81	0.30	0.43	0.59	0.82

- The next step is to determine the ranking of batik SMEs data. At this step weight input is based on Table 6. Using the Interval type-2 FANP method. Before input weight criteria, first determine Rule of criteria SMEs Batik, which is contained in Table 7.

Table 7. Rule of Criteria SMEs Batik

Code	Criteria	Value of Criteria	Description
C1	Batik Variations	a. >=201 variation	Very Good
		b. 101-200 variation	Good
		c. 51-100 variation	Moderate
		d. <=50 variation	Bad
C2	Shop Ownership	a. Privately Owned	Very Good
		b. Join	Good
		c. Rent	Moderate
		d. Seasonal Trader	Bad
C3	Marketplace	a. 3 Market Place	Very Good
		b. 2 Market place	Good
		c. 1 market place	Moderate
		d. No Have Market place	Bad

- The next step is to determine the Linguistic Scales Alternative of Interval Type-2 Trapezoid which is contained in Table 8.

Table 8. Linguistic Scales Alternative of interval Type-2 Trapezoid

Variables of Linguistic	Trapezoid Fuzzy Scale
Bad	[(0, 0.5,1,1.2;1,1) (1.4,1.6,1.8,2;0.8,0,8)]
Moderate	[(1.8,2,3,3.2;1,1) (3.4,3.6,3.8,4;0.8,0,8)]
Good	[(3.8,4,5,5.2;1,1) (5.3,5.6,5.8,6;0.8,0,8)]
Very Good	[(5.8,6,7,7.2;1,1) (7.4,7.6,7.8,8;0.8,0,8)]

- Input of SME data by SME owners is in accordance with Table 9.

Table 9. SMEs Data

SME	Batik Motif Variations	Shop Ownership	Marketplace
Sumber Arafat (A)	Moderate	Good	Very Good
Annisa Batik (B)	Moderate	Good	Good
Bunda Batik (C)	Very Good	Good	Very Good
...	...	...	...
Tia Batik (Z)	Moderate	Very Good	Good

- Then Table 10, conversion to type-2 Fuzzy trapezoid number

Table 10. Convert to type-2 Fuzzy trapezoid number

	C1								C2	C3			
	v_1^u	v_2^{mu}	v_3^{mu}	v_4^u	v_1^L	v_2^{ml}	v_3^{ml}	v_4^L	…	v_1^u	v_2^{mu}	v_3^{mu}	…
A	5.8	6	7	7.2	7.4	7.6	7.8	8	…	5.8	6	7	...
B	1.8	2	3	3.2	3.4	3.6	3.8	4	…	3.8	4	5	...
...	..	...	...	...	...	...	...	...	…	...	...	...	...
Z	1.8	2	3	3.2	3.4	3.6	3.8	4	…	3.8	4	5	...

- After being converted into a trapezoid function, the matrix is normalized and multiplied by the weights derived from FANP. This step is in Table 11 and Table 12.

Table 11. Matrix normalization fuzzy type-2 Trapezoid

	C1								C2	C3				
	v_1^u	v_2^{mu}	v_3^{mu}	v_4^u	v_1^L	v_2^{ml}	v_3^{ml}	v_4^L	…	v_1^u	v_2^{mu}	v_3^{mu}	v_4^u	…
A	0.74	0.77	0.90	0.92	0.95	0.97	1.00	1.03	…	0.73	0.75	0.88	0.90	...
B	0.23	0.26	0.38	0.41	0.44	0.46	0.49	0.51	…	0.48	0.50	0.63	0.65	...
C	0.49	0.51	0.64	0.67	0.68	0.72	0.74	0.77	…	0.73	0.75	0.88	0.90	...
…	...	...	...	...	...	...	...	...	…	..	..	...	...	...
Z	0.12	0.15	0.19	0.26	0.12	0.14	0.19	0.26	…	0.29	0.42	0.59	0.81	...

Table 12. Weighted Normalization Matrix of type-2 FANP

	C1								C2	C3			
	v_1^u	v_2^{mu}	v_3^{mu}	v_4^u	v_1^L	v_2^{ml}	v_3^{ml}	v_4^L	…	v_1^u	v_2^{mu}	v_3^{mu}	…
A	0.09	0.12	0.17	0.24	0.11	0.14	0.19	0.27	…	0.21	0.32	0.52	...
B	0.03	0.04	0.07	0.11	0.05	0.06	0.09	0.13	…	0.14	0.21	0.37	...
…	…												
Z	0.03	0.04	0.07	0.11	0.05	0.06	0.09	0.13	…	0.14	0.21	0.37	...

- The last is Table 13. This is the ranking result of SME Batik Madura.

Table 13. The result of Rank

SMEs	Expert Rank	Trapezoid model-1	Trapezoid model-2	Trapezoid model-3
Sumber Arafat (A)	1	4	3	1
Bunda Batik (C)	2	6	4	2
Annisa Batik (B)	3	7	5	4
...	...	...	...	...
Tia Batik (Z)	200	200	145	200
Accuracy		80.23 %	82%	84%

4. Conclusion

The integration of interval type-2 FANP and type-2 FTOPSIS based on trapezoid fuzzy number is a method for selecting or ranking the best decisions based on many criteria and many alternatives. Type-2 fuzzy trapezium defines membership function in more detail than fuzzy type-1 and type-2 fuzzy triangular. For group-based MCDM, this study uses type-2 fuzzy trapezium geometric mean aggregation. The nonsymmetric trapezoidal type-2 fuzzy membership function can make a significant difference in the results if compared with the fuzzy type-1 membership function. Type-2 fuzzy trapezium can be used optimally by changing the interval points to increase accuracy. The model of the type-2 trapezoid has many variations, so it can be developed for other MCDM methods, such as ELECTRE, VI-KOR, and MACBETH, using different defuzzification methods. Based on trials, the best accuracy at the Trapezoid function with different midpoints and degrees of membership model-3 is 84%.

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