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Identification of plant growth using Bayes Network Base Environmental in Parametric L-System

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ABSTRACT (10PT)

The approaches in the study of plant growth at this time were to use L-system, L-system is a mathematical system for grammatical rules that can be used to describe plants growth. Another extension of L-System is parametric L-System. Parametric L-System is formulated to generate branching tree that can combine the physiological laws in plant growth. The background of this research is environmental factors. The environmental factors can affect plant growth and the factors are dynamic, so that plant growth is dynamic and difficult to approach with certain formulas, where the environmental factor is uncertain for plant growth.
In this paper, we use to propose a new method for an identification of plant growth using Bayes network in parametric L-System based environmental. The novelty of this research using Bayesian Networks to extract structured information from plant growth based environmental. The result of the extract used to generate alphabet, axiom and production rules according to L-System. The result of the research presents a 3-dimensional visualization of plant growth and responsive to the environment change.

I. Introduction

The approach of plant architecture can be categorized as a model plant. The architecture of the plant can be represented in the form of 3D graphics. Aristid Lindenmayer explains growth theory of Anabaena using rewriting system [1] and using grammatical rules as in L-system [2]. The L-system is a method for a model of plant virtual and plant superstructure [3]. The extracting the plant characters use Bayesian Networks, can be used to construct L-system for visual plants growth generating [4]. In this research adding environmental features as fertilizer and implemented in the model of zinnia plant. The extracting the plant characters different in consequence of data retrieval.

To get a visualization of plant growth influenced by the environment be required two L-system-based simulation program, the programs are L-studio and the Virtual Laboratory (Vlab) [1]. The application of L-system method on the virtual agriculture technology analyzing environments for visual plant growth [3]. The environment is a key factor to develop plants. Application of L-system method on plant growth based on environment is done by constructing to model bi-directional information exchange between plants and their environment [5]. The interaction between plants and environment can be exemplified as nutrient plants, plants need nutrients for growth, the nutrients is fertilizer and water for the formation of plant components [6]. The problem in this research is interrelations between fertilizer variations of the organic and inorganic with the growth of plant components, where the change is uncertain.

The plant growth is growing and developing. The grow was the process of increasing the size. The development was the increasing function of body tools. In the plant growth model should be able to create models that accommodate growing and developing, Growth and development were

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interrelated processes. Any changes to the environment will change growth and development in a plant, so, we need a model to identify growth and development in the plant caused by environmental changes. In this study environmental changes are variations in fertilizer organic and inorganic fertilizers. The model to identify growth and development in plant developed with dynamic data according to biological observations and measurements. The model presented a 3-dimensional visualization. The dynamic data were a set of data derived from environmental and plant data. The dynamic database used to develop a model in L-system as image synthesis. The image synthesis generated from turtle interpretation of L-systems and a programming language based on L-systems.

Bayes Network can solve problems that are dynamic [7], in this research use Bayes network because Bayes network gives the flexibility to represent data structures, in this case, the data structure is the growth of zinnia plants. Bayes Network method was simple Probabilistic Graphical Model (PGM), Bayes Network constructed from probability theory and graph theory. The probability theory is related to data, while graph theory is the representation to be obtained [8]. In this research, Bayes network can be used to modeling value of zinnia plant component and variation of fertilizer. Bayes Network can be used as a tool for prediction variation of fertilizer with plant growth. The model of zinnia plant growth consists of the qualitative model and quantitative model when constructing a model of plant growth in parametric L-system [9].

II. Method

Preparation of this paper using experimental methods, experimental methods were methods performed by manipulating objects and control. The number of plants is fifty as training data and six as test data. The research implemented by first preparing zinnia plants with age, weight, and size was same on the planting medium. Each pot planted zinnia. Then each pot was given the same treatment for watering, light intensity, and other environmental factors. In the research, an environmental influence was the treatment fertilizer. The fertilizer is a variation of organic and inorganic. Once the stems of the zinnia plant begin to grow. The stem length was measured using a ruler. The measured growth and development indicators are sprout as height, stalk as height, leaf area as length x width, and flower area as diameter. The grow and develop data of zinnia plant obtained during observation for twenty-five days. On every day measurements are done every at 8 am. Flow diagram for zinnia plant growth model in this research can be seen in figure 1.

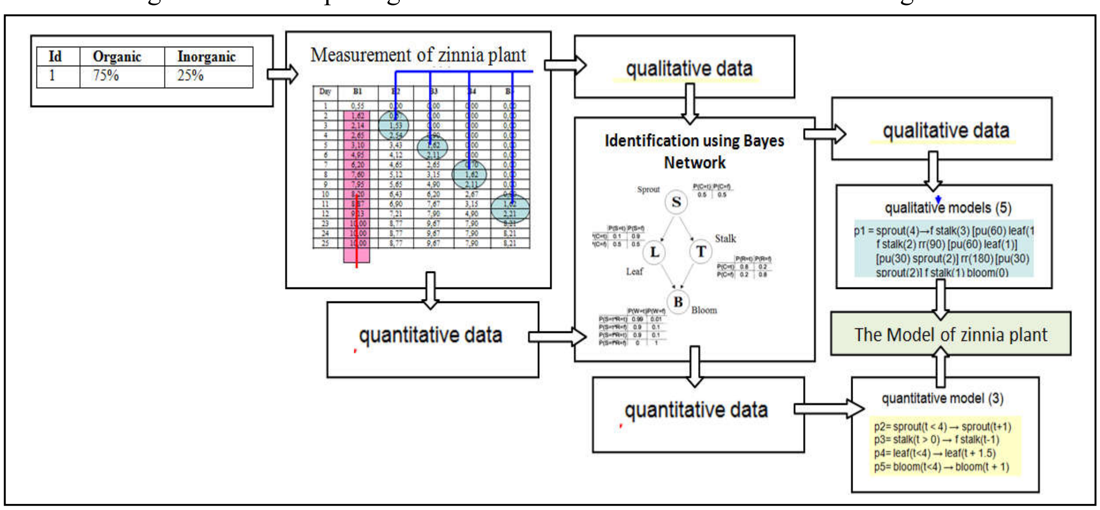

Fig. 1. Flow diagram model of zinnia plant

In figure 4, L-System used to built of visual zinnia plant. In this research, this diagram has six stages, (1) the data was collected from fertilizer treatment variation,(2) the data was collected from measured growth of zinnia plant, (3) the process build function of plant growth, (4) the process build the quantitative model, (5) the process build syntax grammar from the quantitative model, (6) the process build the qualitative model, (7) the process build syntax grammar from the qualitative model, (8) the process build model of zinnia plant.

III. Results and Discussion

In figure 4, L-System used to built of visual zinnia plant. In this research, this diagram has six stages, (1) the data was collected from fertilizer treatment variation,(2) the data was collected from measured growth of zinnia plant, (3) the process build function of plant growth, (4) the process build the quantitative model, (5) the process build syntax grammar from the quantitative model, (6) the process build the qualitative model, (7) the process build syntax grammar from the qualitative model, (8) the process build model of zinnia plant.

A. Visual Models of Plants

The research of plant model began when Aristid Lindenmayer explain the theory of cell growth in Anabaena Catenula with string rewriting, the theory was Lindenmayer System (L-system) [1], later the theory improved to higher plants model, complex branching structures, in particular, inflorescences, and modular in plant [2]. L-System is used on various types of plants [3]. In L-systems, The module in zinnia plant will develop separately dan repeated for example, a stalk, leaf, a flower, or a branch, the visual model of plant follow in figure 2

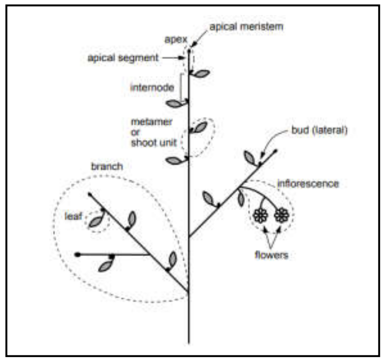

Fig. 2. The modular in plant [3].

B. Lindenmayer System (L-system)

L-system is a parallel rewriting system and a type of formal grammar, an L-system consists of an alphabet of symbols, each symbol into some string used represent plants growth, an axiom of string use to begin construction, an example from a graphics object defined by rewriting rule is the snowflake curve, in 1905 by von Koch [1]. The rewriting rule consists of two arts, that parts were initiator and generator,  used as initiator,  used as generator, after the one iteration is  , the one iteration replaces each line in initiator with a generator can follow in figure 3.

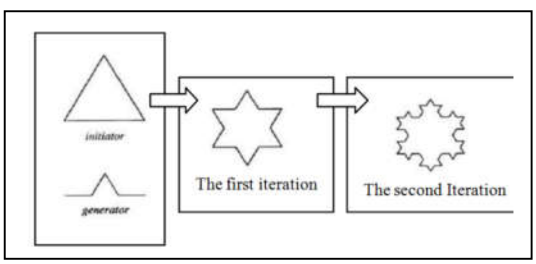

Fig. 3. An iterative illustration of the snowflake curve

L-systems with turtle interpretation can generate a variety of visualization plants. A turtle can move in any direction, forward, backward, rightward, or leftward. The symbol with letter f represents commands to move forward the turtle; the symbol with letter b represents commands to move backward the turtle. The symbol in L-system encodes turtle geometry for generating graphical representations of the leaves, blooms, and stalks. One-symbol will translate one string into a 3D geometrical object resembling a plant can follow in table 1.

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Table 1. Symbols used in plant growth

Symbol of string	Description
leaf[s]	Advances the turtle by a step size of s in the plant leaves.
bloom[s]	Advances the turtle by a step size of s in the plant flowers.
stalk [s]	Advances the turtle by a step size of s in the direction right
pu(α)	The turtle is tilted up around axis y and an angle of α degree.
pd(α)	The turtle is tilted down around axis y and an angle of α degree.
rr(α)	The turtle is rotated clockwise around axis x and an angle of α degrees.

C. Parametric L-Systems

Parametric L-system is a module in L-System. A symbol a can be associate with any number of real-valued a_1, a2... $a_n \in$ Real, written in parenthesis as A (a_1, a_2... an), Additionally, parameters may be combined with other parameters or numeric constants in arithmetic expressions involving common operators like +, −, ∗, / and the exponentiation operator ˆ, mathematical functions like sine, cosine, tangent, arc cosines, arc sine, arc tan, floor, ceiling, truncate, absolute value, exponential, logarithms as well as a random function. Productions in a parametric L-system may specify a condition that must be met in order for the production to be applied. A condition C consists of logical expressions that may combine parameters or arithmetic expressions via relational operators such as >, <, ≥, ≤, ==,!= that evaluate to Boolean values, and may themselves be combined via logical operators && and ||, the axiom is used to start production, a syntax grammar for axiom is ω.

A module in L-System has a left and a right context, given by the modules to its left and to its right. Parametric L-system is also referred to as 1L-systems or 2L-systems. The predecessors of a production in a 1L-system have only one definition of production in one context. The context in the predecessor can product as predecessor → successor. The predecessor of a production in a 2L-system may specify a left and/or right context as strings of modules. The context in the predecessor can product as predecessor: condition → successor, rules of production in parametric L-system are two rules, the first production was a production to one context, and there is a syntax grammar as b → a, the meaning was the letter a can produce the letter b. The second production was a production to more one context, there is the syntax b < a and b → a, the meaning is the letter a can produce the letter b if and only if the condition is a letter a is less than the letter b, the illustration can follow in figure 4.

ω : baaaaaa
P_1 : b < $a \rightarrow b$
P_2 : b → a
Iteration 1 : baaaaaa
Iteration 2 : abaaaaa
Iteration 3 : aabaaaa

Fig. 4. The production rules in Parametric L-system [1]

In this research, the software development for visualisation of plant using kLSystems package in MathEvolvica [10], the kLSystems package contains application of rewrite rules in L-Systems, the production rule is the syntax $l < p > r \rightarrow s$, the production rule using kLSystems package is LRule[LEFT[l], PRED[p], RIGHT[r], SUCC[s]]. The KLSystem package is easy to perform expressions and subexpressions only see the multi-level symbol. kLSystems package in MathEvolvica can be used to build virtual of zinnia plant. The virtual of zinnia plant describing growth sequences of sprouts, leaves, and blooms based graphical

D. Bayesian Network Approach

Bayesian network is a graphic model for represents an interaction between variables. The Bayesian network can be described as a graph; the graph consists of a node (node) and an arc (arc). The node shows the variable, such as x, then the probability value x is p(x), an arc shows interrelationship node. If there are relationships from node X to node Y, then the relationship is variable X influence on variable Y, the influence can be expressed as a conditional probability. In the Bayesian network, the relationship can be mutually dependent (related). The corresponding directed acyclic graph is depicted in figure 5, there are three dependent variables, the variables to be affected is sprout, the variables affect is organic and inorganic,

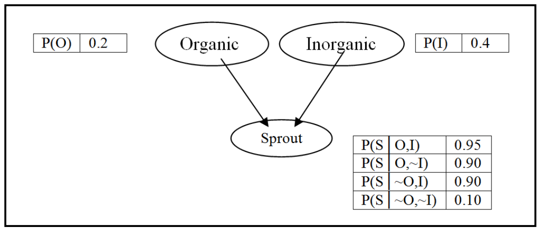

Fig. 5. Directed acyclic graph representing two independent possible causes of a sprout growth.

In figure 5, in this simplified illustration, we assume only two possible causes of this sprout growth: organic and inorganic. As an example search probability of sprout if organic is true, then the probability of organic is after known sprout can follow equation (1).

$$P(S|O) = P(S|I,O)P(I|O) + P(S|{\sim}I,O)P({\sim}I|O) = 0.92 \quad (1)$$

E. Joint probability distribution.

The joint probability distribution is a probability distribution has two random variables, two random variables are x and y. The probability distribution that defines their simultaneous behavior.The probability distribution can be represented as a function for all pairs (x, y) [11].

In the table 2 can be taken that for two discrete random variables is $P(X = x, Y = y)$. Measurements for the length and width of a leaf area are the nearest mm. Let X denotes the length and Y denotes the width. The values of X are 132, 133, and 134 mm. The values of Y are 17 and 18 mm. There are the six of pairs (X, Y). The joint probability distribution for each pair can follow in table 2.

Table 2. Symbols used in plant growth

		x=length		
		132	***133***	***134***
y=width	***17***	0.12	0.42	0.06
	18	0.08	0.28	0.04

The sum of all the probabilities is 1.0. The highest probability of all pairs is (133, 17). The lowest probability of all pairs is (132, 17). The joint probability function is the function $f_{XY}(x, y) = P(X = x, Y = y)$.

F. Qualitative Model

The process builds qualitative models is identify sequences develop in zinnia plant. The sequences were a sprout, leaves, and bloom. In L-system, the first syntax is an axiom, the axiom consists module A describing the sprout. After some time, the sprout will develop two internodes with a symbol I, two lateral leaves with symbol L and two lateral sprouts, a bloom bud with symbol K. The main sprout is not continued for developing. The development of the sprout until the bloom is a module. The apical growth is continued according to the same pattern on the lateral sprout. Over time, the size of the leaves increases and the bloom buds change into fully developed flowers. The two lateral sprout develop at different rates. In development is asymmetric after inflorescence overall. The model of zinnia plant growth in L-system can follow in figure 6.

$\omega: A_4$
$P_1 : A_4 \rightarrow I_2[L_3\ A_2][\ L_2\ A_3]I_1K_0$
$P_2 : A_i \rightarrow A_i{+}1,\ 0 \leq i < 4$
$P_3 : L_i \rightarrow L_i{+}1,\ 0 \leq i < 4$
$P_4 : I_i \rightarrow I_i{+}1,\ i \geq 0$
$P_5 : K_i \rightarrow K_i{+}1,\ i \geq 0$

Fig. 6. The model of zinnia plant growth in L-system

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The model qualitative of zinnia plant grow is a development model that connects each plant component. the plant components are the bud, the stem, the leaf, and the flower. The relationship can build a set. The set can be denoted as $G_{zinnia}=(\Sigma, P=\{P_1,\ldots,P_9\}, \alpha)$, then alphabet of Σ is $\Sigma = (\delta_1, ..., \delta_n)$, every symbol of the alphabet represent morphological units as sprouts, stalk, leaf, flower, alphabet of α is beginning of a string, alphabet of α called the axiom, alphabet of P is $P = (P_1, ..., P_n)$, alphabet of P is a set of productions. The alphabet of P_1 is the first production, the first production is an explanation about the relationship between the plant components, for the bud notated the sprout, the stem notated the stalk, the leaf notated leaf and then the flower notated bloom. The first production can be called The model qualitative of zinnia plant grow. The first production form grammar rule is P_1 = sprout(4) → stalk(2) [leaf(3) sprout(2)] [leaf(2) sprout(3)]] stalk(1) bloom(0). The Structure of zinnia plant qualitative models in L-System can follow in figure 7.

$G_{Zinnia}=(\Sigma, P=\{P_1,\ldots,P_9\}, \alpha)$
Σ= {pd, pu, rr, sprout, stalk, leaf, bloom}
α = sprout (4)
P = Production is the relationship of plant components
P_1 = sprout(4) → stalk(2) [leaf(3) sprout(2)] [leaf(2) sprout(3)]] stalk(1) bloom((

Fig. 7. The structure of zinnia plant qualitative models in L-System.

G. Quantitative Model

The qualitative models in the growth of zinnia plant is a model to determine the growth function of every component. The growth function is obtained during observation. The every of plant component will be obtained the growth function. The plant components are the sprout, the stalk, the leaf, and then bloom. In this research, mathematics formula for growth function [5] can follow in (1). The result of stalk growth function have form graph and comparison between real with a result of function growth can follow in figure 8. In Table obtained value of growth function parameter for the sprout, stalk, leaf, and bloom.

$$G(t) = L + \frac{U - L}{1 + e^{m(T-t)}} \quad (1)$$

Where : L: The minimum value; U : The maximum value; m : slope (the estimated value of observation data); T: time in (U-L) / 2; t : time variable.

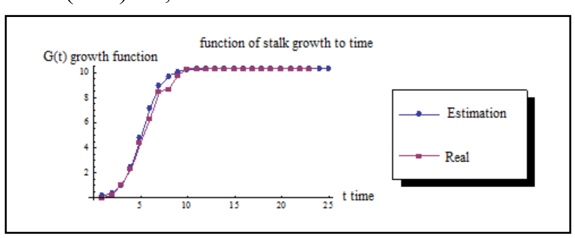

Fig. 8. The graph of growth function result in stalk

In Table 3, Each of plant component will be controlled by the growth function. In stalk notation used production two, The value of stalk growth for time maximum four, so the stalk growth in the sequence are stalk(1), stalk(2), stalk(3), and stalk(4). In leaf notation used production three, The time maximum value of leaf growth is four, so the leaf growth in the sequence is the leaf (1), leaf (2), leaf (3), and leaf (4). The equal formula is for bloom and sprout. The structure of zinnia plant quantitative models in L-System can follow in 9.

Table 3. The value of growth function parameter for sprout, stalk, leaf ,and bloom

Notation	**L**	**U**	**m**	**T**
Stalk	0	10.3	0.9	4.4
Leaf	0	6.6	0.9	4.1
Bloom	0	7.2	0.8	4
Sprout	1	8	0.8	4

P_{2}= sprout (t<4) ⟶ sprout (t+1)
P_{3}= stalk (t<4) ⟶ stalk (t+1)
P_{4}= leaf (t<4) ⟶ leaf (t+1)
P_{5}= bloom(t<4) ⟶ bloom(t+1)

Fig. 9. The structure of zinnia plant quantitative models in L-System.

H. Bayesian network approach

In this research, Bayesian network approach used for identification of parameter in the growth of zinnia plant and variation of organic and inorganic fertilizer. The result of Bayesian network approach used to input in a model of zinnia plant. The process of the Bayesian network is three stages.

The first stage makes directed acyclic graph for the growth of zinnia plant-based environmental in figure 10. The directed acyclic graph developed using Samlam software. The node is random variables growth of zinnia plant in table 4, growth of zinnia plant in table 5 and fertilizer in table 6, the form of a node is oval, the arrow is connected pairs of the node (organic is a parent of sprout if there is an arrow from organic node to sprout node).

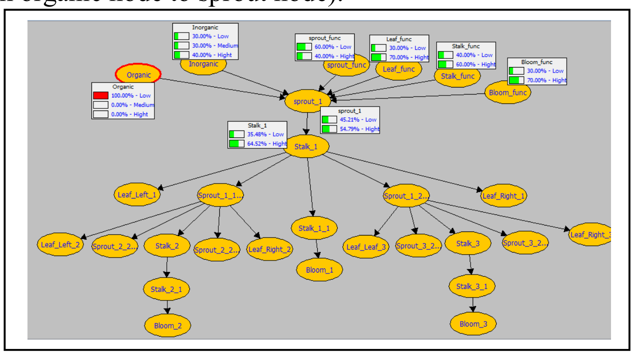

Fig. 10. Directed graphical model representing two independent potential causes of growth sprout with the prior probability distribution.

Table 4. The variables growth of zinnia plant in quantitative data

Id	The code of plant component	Name of plant component	Composition	Category
1	Sprout function	The function of growth sprout	≤ 4	Low
			>4	High
2	Stalk function	The function of growth stalk	≤ 4	Low
			>4	High
3	Leaf	The function of growth leaf	≤ 4	Low
			>4	High
4	Bloom	The function of growth bloom	≤ 4	Low
			>4	High

Table 5. The variables of zinnia plant growth in qualitative data

Id	The code of plant component	Name of plant component
1	Sprout 1	The first sprout
2	Stalk 1	The first Stalk
3	Leaf left 1	The first leaf left
4	Sprout 1.1	The sub first sprout
5	Stalk 1.1	The sub first Stalk
6	Bloom 1	The first flower
7	Sprout 1.2	The sub second sprout
8	Leaf right 1	The first right

Table 6. The variables of fertilizer

Id	The code of fertilizer	Name of fertilizer	Composition	Category
1	Organic	The organic fertilizer	0-25 gram	Low
			26-50 gram	Medium
			51-75 gram	High
2	Inorganic	The Inorganic fertilizer	0-25 gram	Low
			26-50 gram	Medium
			51-75 gram	High

The second stage, the number of plants is fifty as training data. The component of zinnia plant given a value of probability, the component of zinnia plant is divided into two parts. The two parts are qualitative data and quantitative data. The qualitative data can follow in table 4. In table 4, the qualitative data as the function of growth sprout is a middle time between minimum time and maximum time as sprout grow. The value of growth sprout divided by two i.e. low and high. The low value is <4, and the high value is ⩾ 4. The quantitative data can follow in table 5. The variation of organic and inorganic fertilizer as environmental variable given a value of probability in table 6. In table 6, the fertilizer variable of organic is a number of fertilizer compositions for one plant. The value composition of organic fertilizer divided by three i.e. low, medium, and high. The low value is < 25 gram, the medium value is 26-50 gram, and the high value is 51-75 gram. The third stage entering the probability of zinnia plant component and the probability of fertilizer in SamIam software. The SamIam software used to calculate the conditional probability. The result of design using SamIam software can explain probability for every component of zinnia plant in the node. In figure 10, example if the value of organic fertilizer is < 25 then the value of stalk is high; the probability is 64, 52%.

I. Model of zinnia plant and Visualization

Model of zinnia plant combined qualitative and quantitative models. The model contains develop and growth on zinnia plant can follow in figure 11.

$G_{zinnia}=(\Sigma, P=\{P_1,\ldots,P_9\},\alpha)$
$\Sigma=$ {pd, pu, rr, sprout, stalk, leaf, bloom}
$\alpha =$ sprout (4)
P = Production is the relationship of plant components
$P_1 =$ sprout(4) $\rightarrow$ stalk(2) [leaf(3) sprout(2)] [leaf(2) sprout(3)]] stalk(1) bloom(0)
$P_{2=}$ sprout (t<4) $\rightarrow$ sprout (t+1)
$P_{3=}$ stalk (t<4) $\rightarrow$ stalk (t+1)
$P_{4=}$ leaf (t<4) $\rightarrow$ leaf (t+1)
$P_5=$ bloom(t<4) $\rightarrow$ bloom(t+1)

Fig. 11. Structure model of zinnia plant in L-System.

The package MathEvolvica used visual of plant growth, the package consists kLSystem.m and TurtleInterpretation.m function. The kLSystem.m function used for initialization in L-System. The development and growth of zinnia plant are decoded in Mathematica software. The TurtleInterpretation.m function used in the visualization of zinnia plant model. The Mathematica program code will be converted into a computer graphic.

The visualization of zinnia plants growth with six iterations can follow in figure Figure 12. In Figure 12 number 1, the first iteration of the grammar rule is replacing sprout with two internodes, two lateral leaves, two lateral sprouts, and a bloom bud. In Figure 12 number 2, the second iteration of the grammar rule accord with function growth. In Figure 12 number 3, the next process continues in the first iteration is said to be a rewriting rule.

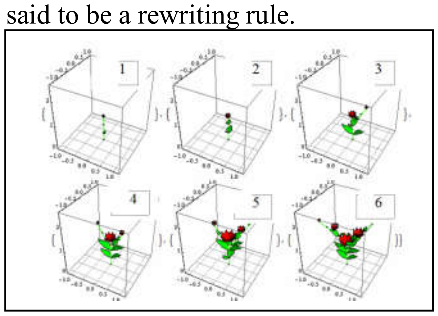

Fig. 12. The visualization growth of zinnia plant with six iterations

The accuracy of zinnia plant model can be determined by counting error rate in table 7, The error rate is generated compare between real with visuals in plant height. To count error rate, in this research using Mean Absolute Percentage Error (MAPE), with mathematics equation (2).

$$\text{MAPE} = \frac{\sum \frac{(x_i - y_i)}{x_i} x\ 100\%}{n} \quad (2)$$

With Xi is actual data number-i and Yi is visual data number-i. The calculation results were obtained an average level of error percentage is under 10 %. Level of error rate (MAPE) less than 40% can be said as good and dependable[12].

Table 7. The comparison of zinnia plant hight between real and visual

Plant Code	**The zinnia plant hight (cm)**		
	Real	*Visual*	*Error*
1	28.6	24.3	4.3
2	24.8	29.7	4.9
3	29.3	24	5.3
4	24.7	26.8	2.1
5	23.1	28	4.9
6	22.4	27.8	5.4
Mean	25.4	26.7	4.4

IV. Conclusion

In this research, the identification growth of zinnia plant developed using Bayesian network based data observation in the field. The results of the Bayesian network can identification visualization growth of zinnia plant model. The model can be demonstrated about the difference of plant height between real and visual is less than 10 % on average. The weakness of this research is a little zinnia plant data. The visualization model of zinnia plant shows a response to fertilizer changes. It is concluded the zinnia plant agriculture can develop further with visualization growth of zinnia plant base environmental.

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